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258. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum the infinite series $\frac{n^2}{(4n^2-1)^2}$ beginning with $n=1$, n being always odd.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\left[\frac{1}{(2n-1)^2}+\frac{1}{(2n+1)^2}-\frac{1}{2n+1}+\frac{1}{2n-1}\right]$$

Let $u=1, 3, 5, 7,$ .... Then

$$\Sigma\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+.....\right]+\tfrac{1}{16}\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-.........\right].$$

$$\therefore\ \Sigma\frac{n^2}{(4n^2-1)^2}=\tfrac{1}{16}\cdot\frac{\pi^2}{8}+\tfrac{1}{16}\cdot\frac{\pi}{4}=\frac{\pi}{64}[\tfrac{1}{2}\pi+1]\quad \text{(See line 1, p. 41).}$$

Also solved by G. W. Greenwood, and Henry Heaton.

CALCULUS.

133. Proposed by NELSON L. RORAY, South Jersey Institute, Bridgeton, N. J.

Evaluate $\int\frac{\sqrt{(1+y)}}{1+y^2}dy$.

Solution by HENRY HEATON, Belfield, N. D.

Let $1+y=z^2$; then $\int\frac{\sqrt{(1+y)}}{1+y^2}dy=\int\frac{2z^2dz}{z^4-2z^2+2}=\frac{1}{2a}\int\left(\frac{z}{z^2-2az+\sqrt{2}}-\frac{z}{z^2+2az+\sqrt{2}}\right)dz$, where $a=\sqrt{\frac{\sqrt{2}+1}{2}}$.

$$\frac{1}{2a}\int\left(\frac{z}{z^2-2az+\sqrt{2}}-\frac{z}{z^2+2az+\sqrt{2}}\right)dz=\frac{1}{2a}\int\left(\frac{(z-a)+a}{z^2-2az+\sqrt{2}}\right.$$

$$\left.\frac{(z+a)-a}{z^2-2az+\sqrt{2}}\right)dz=\frac{1}{2a}\log\sqrt{\frac{z^2-2az+\sqrt{2}}{z^2+2az+\sqrt{2}}}+\frac{1}{2\sqrt{[\sqrt{2}-a^2]}}\left(\tan^{-1}\frac{z-a}{\sqrt{[\sqrt{2}-a^2]}}\right.$$

$$\left.+\tan^{-1}\frac{z+a}{\sqrt{[\sqrt{2}-a^2]}}\right)=\sqrt{\frac{\sqrt{2}-1}{2}}\log\sqrt{\frac{1+y-\sqrt{[2\sqrt{2}+2]}\sqrt{[1+y]}+\sqrt{2}}{1+y+\sqrt{[2\sqrt{2}+2]}\sqrt{[1+y]}+\sqrt{2}}}$$

$$+\sqrt{\frac{\sqrt{2}+1}{2}}\tan^{-1}\left(\frac{\sqrt{[2\sqrt{2}-2]}\sqrt{[1+y]}}{\sqrt{2}-1-y}\right).$$

The above solution is in the real factors of z^4-2z^2+2.

On page 140, No. 5, Vol. IX, Dr. Zerr gives a solution, using the imaginary factors of z^4-2z^2+2. His final seemingly imaginary result reduces to a real value by substituting $\sqrt{\frac{1+\sqrt{2}}{2}}+\sqrt{\frac{1-\sqrt{2}}{2}}$ for $\sqrt{[1+\sqrt{(-1)}]}$ and $\sqrt{\frac{1+\sqrt{2}}{2}}-\sqrt{\frac{1-\sqrt{2}}{2}}$ for $\sqrt{[1-\sqrt{(-1)}]}$, reducing and remembering that

$$\log[a+b\sqrt{(-1)}]=\sqrt{(-1)}\tan^{-1}\frac{b}{a}+\log\sqrt{(a^2+b^2)}.$$

216. **Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.**

Find the limit of the sum of the series

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots\ldots+\frac{n}{n^2+m^2},$$

when n and m are indefinitely increased. (Distinguish the several cases arising from the different *relative* values of m and n.)

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+\ldots\ldots+\frac{n}{n^2+m^2}$$

$$=h\left[1+\frac{1}{1+h^2}+\frac{1}{1+(2h)^2}+\frac{1}{1+(3h)^2}+\ldots\ldots+\frac{1}{1+(mh)^2}\right]$$

where $h=\frac{1}{n}$. Hence the required limit is $\int_0^{mh}\frac{dx}{1+x^2}=\tan^{-1}(mh)$.

If m is small compared with n so that $mh=0$, the limit is $\tan^{-1}0=0$. If m is equal to n so that $mh=1$, the limit is $\tan^{-1}1=\frac{1}{4}\pi$. If m is large compared with n so that $mh=\infty$, the limit is $\tan^{-1}\infty=\frac{1}{2}\pi$.

Also solved by S. A. Corey, and Henry Heaton.

217. **Proposed by S. A. COREY, Hiteman, Iowa.**

In *The Analyst*, Vol. II, p. 120, 1875, Dr. G. W. Hill finds by the method of mechanical quadrature the value of $\int_0^{\frac{1}{2}\pi}\frac{xdx}{\sin x[1+.16\cos^2x]^{\frac{3}{2}}}$ to be 1.6576363.

Evaluate the definite integral by some other method and verify above result.

Solution by the PROPOSER.

Using the formula* $f(x)-f(0)=\frac{x}{m.2}\left\{[f'(x)+f'(0)]+2\left[f'\left(\frac{x}{m}\right)\right.\right.$

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*Monthly, Vol. 13, p. 12.